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Maxima 5.9.1 http://maxima.sourceforge.net
Using Lisp CMU Common Lisp 19a
Distributed under the GNU Public License. See the file COPYING.
Dedicated to the memory of William Schelter.
This is a development version of Maxima. The function bug_report()
provides bug reporting information.

(%i1) load("tensor.lisp");

(%o1) tensor.lisp

(%i2) a : [1,2] ot [sin(x),cos(x),tan(x)] ot [1,x,x^2,x^4];

(%o2) [[[sin x, x sin x, x2 sin x, x4 sin x], [cos x, x cos x, x2 cos x, x4 cos x], [tan x, x tan x, x2 tan x, x4 tan x]], [[2 sin x, 2 x sin x, 2 x2 sin x, 2 x4 sin x], [2 cos x, 2 x cos x, 2 x2 cos x, 2 x4 cos x], [2 tan x, 2 x tan x, 2 x2 tan x, 2 x4 tan x]]]

(%i3) tensortype(a);

(%o3) [2,3,4]

(%i4) tensorrank(a);

(%o4) 3

(%i5) b : [s,t];

(%o5) [s,t]

(%i6) c : a ot b;

(%o6) [[[s sin x, t sin x], [s x sin x, t x sin x], [s x2 sin x, t x2 sin x], [s x4 sin x, t x4 sin x]], [[s cos x, t cos x], [s x cos x, t x cos x], [s x2 cos x, t x2 cos x], [s x4 cos x, t x4 cos x]], [[s tan x, t tan x], [s x tan x, t x tan x], [s x2 tan x, t x2 tan x], [s x4 tan x, t x4 tan x]]], [[[2 s sin x, 2 t sin x], [2 s x sin x, 2 t x sin x], [2 s x2 sin x, 2 t x2 sin x], [2 s x4 sin x, 2 t x4 sin x]], [[2 s cos x, 2 t cos x], [2 s x cos x, 2 t x cos x], [2 s x2 cos x, 2 t x2 cos x], [2 s x4 cos x, 2 t x4 cos x]], [[2 s tan x, 2 t tan x], [2 s x tan x, 2 t x tan x], [2 s x2 tan x, 2 t x2 tan x], [2 s x4 tan x, 2 t x4 tan x]]]]

(%i7) tensortype(c);

(%o7) [2,3,4,2]

(%i8) d : b ot a;

(%o8) [[[s sin x, s x sin x, s x2 sin x, s x4 sin x], [s cos x, s x cos x, s x2 cos x, s x4 cos x], [s tan x, s x tan x, s x2 tan x, s x4 tan x]], [[2 s sin x, 2 s x sin x, 2 s x2 sin x, 2 s x4 sin x], [2 s cos x, 2 s x cos x, 2 s x2 cos x, 2 s x4 cos x], [2 s tan x, 2 s x tan x, 2 s x2 tan x, 2 s x4 tan x]]], [[[t sin x, t x sin x, t x2 sin x, t x4 sin x], [t cos x, t x cos x, t x2 cos x, t x4 cos x], [t tan x, t x tan x, t x2 tan x, t x4 tan x]], [[2 t sin x, 2 t x sin x, 2 t x2 sin x, 2 t x4 sin x], [2 t cos x, 2 t x cos x, 2 t x2 cos x, 2 t x4 cos x], [2 t tan x, 2 t x tan x, 2 t x2 tan x, 2 t x4 tan x]]]]

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(%i9) tensortype(d);

(%o9) [2,2,3,4]

(%i10) e : [-1, 2, 0, 3];

(%o10) [-1,2,0,3]

(%i11) tensortype(e);

(%o11) [4]

(%i12) load(eigen);

Warning - you are redefining the MACSYMA function EIGENVALUES
Warning - you are redefining the MACSYMA function EIGENVECTORS
(%o12) /usr/local/share/maxima/5.9.1/share/matrix/eigen.mac

(%i13) f : d tvc e;

(%o13) [[[ (3 s x4 + 2 s x - s) sin x, (3 s x4 + 2 s x - s) cos x, (3 s x4 + 2 s x - s) tan x], [(6 s x4 + 4 s x - 2 s) sin x, (6 s x4 + 4 s x - 2 s) cos x, (6 s x4 + 4 s x - 2 s) tan x]], [[(3 t x4 + 2 t x - t) sin x, (3 t x4 + 2 t x - t) cos x, (3 t x4 + 2 t x - t) tan x], [(6 t x4 + 4 t x - 2 t) sin x, (6 t x4 + 4 t x - 2 t) cos x, (6 t x4 + 4 t x - 2 t) tan x]]]

(%i14) tensortype(f);

(%o14) [2,2,3]

(%i15) g : d ot e;

(%o15) [[[[[-s sin x, 2 s sin x, 0, 3 s sin x], [-s x sin x, 2 s x sin x, 0, 3 s x sin x], [-s x2 sin x, 2 s x2 sin x, 0, 3 s x2 sin x], [-s x4 sin x, 2 s x4 sin x, 0, 3 s x4 sin x]], [[-s cos x, 2 s cos x, 0, 3 s cos x], [-s x cos x, 2 s x cos x, 0, 3 s x cos x], [-s x2 cos x, 2 s x2 cos x, 0, 3 s x2 cos x], [-s x4 cos x, 2 s x4 cos x, 0, 3 s x4 cos x]], [[-s tan x, 2 s tan x, 0, 3 s tan x], [-s x tan x, 2 s x tan x, 0, 3 s x tan x], [-s x2 tan x, 2 s x2 tan x, 0, 3 s x2 tan x], [-s x4 tan x, 2 s x4 tan x, 0, 3 s x4 tan x]], [[[-2 s sin x, 4 s sin x, 0, 6 s sin x], [-2 s x sin x, 4 s x sin x, 0, 6 s x sin x], [-2 s x2 sin x, 4 s x2 sin x, 0, 6 s x2 sin x], [-2 s x4 sin x, 4 s x4 sin x, 0, 6 s x4 sin x]], [[-2 s cos x, 4 s cos x, 0, 6 s cos x], [-2 s x cos x, 4 s x cos x, 0, 6 s x cos x], [-2 s x2 cos x, 4 s x2 cos x, 0, 6 s x2 cos x], [-2 s x4 cos x, 4 s x4 cos x, 0, 6 s x4 cos x]], [[-2 s tan x, 4 s tan x, 0, 6 s tan x], [-2 s x tan x, 4 s x tan x, 0, 6 s x tan x], [-2 s x2 tan x, 4 s x2 tan x, 0, 6 s x2 tan x], [-2 s x4 tan x, 4 s x4 tan x, 0, 6 s x4 tan x]]], [[[[-t sin x, 2 t sin x, 0, 3 t sin x], [-t x sin x, 2 t x sin x, 0, 3 t x sin x], [-t x2 sin x, 2 t x2 sin x, 0, 3 t x2 sin x], [-t x4 sin x, 2 t x4 sin x, 0, 3 t x4 sin x]], [[-t cos x, 2 t cos x, 0, 3 t cos x], [-t x cos x, 2 t x cos x, 0, 3 t x cos x], [-t x2 cos x, 2 t x2 cos x, 0, 3 t x2 cos x], [-t x4 cos x, 2 t x4 cos x, 0, 3 t x4 cos x]], [[-t tan x, 2 t tan x, 0, 3 t tan x], [-t x tan x, 2 t x tan x, 0, 3 t x tan x], [-t x2 tan x, 2 t x2 tan x, 0, 3 t x2 tan x], [-t x4 tan x, 2 t x4 tan x, 0, 3 t x4 tan x]]], [[[[-2 t sin x, 4 t sin x, 0, 6 t sin x], [-2 t x sin x, 4 t x sin x, 0, 6 t x sin x], [-2 t x2 sin x, 4 t x2 sin x, 0, 6 t x2 sin x], [-2 t x4 sin x, 4 t x4 sin x, 0, 6 t x4 sin x]], [[-2 t cos x, 4 t cos x, 0, 6 t cos x], [-2 t x cos x, 4 t x cos x, 0, 6 t x cos x], [-2 t x2 cos x, 4 t x2 cos x, 0, 6 t x2 cos x], [-2 t x4 cos x, 4 t x4 cos x, 0, 6 t x4 cos x]], [[-2 t tan x, 4 t tan x, 0, 6 t tan x], [-2 t x tan x, 4 t x tan x, 0, 6 t x tan x], [-2 t x2 tan x, 4 t x2 tan x, 0, 6 t x2 tan x], [-2 t x4 tan x, 4 t x4 tan x, 0, 6 t x4 tan x]]]]]

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(%i16) tensortype(g);

(%o16) [2, 2, 3, 4, 4]

(%i17) ff : cntrlst2(g);

(%o17) [[[3 s x4 sin x + 2 s x sin x - s sin x, 3 s x4 cos x + 2 s x cos x - s cos x, 3 s x4 tan x + 2 s x tan x - s tan x], [6 s x4 sin x + 4 s x sin x - 2 s sin x, 6 s x4 cos x + 4 s x cos x - 2 s cos x, 6 s x4 tan x + 4 s x tan x - 2 s tan x]], [[3 t x4 sin x + 2 t x sin x - t sin x, 3 t x4 cos x + 2 t x cos x - t cos x, 3 t x4 tan x + 2 t x tan x - t tan x], [6 t x4 sin x + 4 t x sin x - 2 t sin x, 6 t x4 cos x + 4 t x cos x - 2 t cos x, 6 t x4 tan x + 4 t x tan x - 2 t tan x]]]

(%i18) tensortype(ff);

(%o18) [2, 2, 3]

(%i19) ff[2][1][3];

(%o19) 3 t x4 tan x + 2 t x tan x - t tan x

(%i20) ratsimp(f - ff);

(%o20) [[[0, 0, 0], [0, 0, 0]], [[0, 0, 0], [0, 0, 0]]]

(%i21) fff : cntrct(g, 4, 5);

(%o21) [[[3 s x4 sin x + 2 s x sin x - s sin x, 3 s x4 cos x + 2 s x cos x - s cos x, 3 s x4 tan x + 2 s x tan x - s tan x], [6 s x4 sin x + 4 s x sin x - 2 s sin x, 6 s x4 cos x + 4 s x cos x - 2 s cos x, 6 s x4 tan x + 4 s x tan x - 2 s tan x]], [[3 t x4 sin x + 2 t x sin x - t sin x, 3 t x4 cos x + 2 t x cos x - t cos x, 3 t x4 tan x + 2 t x tan x - t tan x], [6 t x4 sin x + 4 t x sin x - 2 t sin x, 6 t x4 cos x + 4 t x cos x - 2 t cos x, 6 t x4 tan x + 4 t x tan x - 2 t tan x]]]

(%i22) ratsimp(ff - fff);

(%o22) [[[0, 0, 0], [0, 0, 0]], [[0, 0, 0], [0, 0, 0]]]

(%i23) tensortype(fff);

(%o23) [2, 2, 3]

(%i24) cntrct(fff, 1, 2);

(%o24) [6 t x4 sin x + 3 s x4 sin x + 4 t x sin x + 2 s x sin x - 2 t sin x - s sin x, 6 t x4 cos x + 3 s x4 cos x + 4 t x cos x + 2 s x cos x - 2 t cos x - s cos x, 6 t x4 tan x + 3 s x4 tan x + 4 t x tan x + 2 s x tan x - 2 t tan x - s tan x]

(%i25) tensortype(%o24);

(%o25) [3]

(%i26) kill(a,b,c,d,e,f,ff,fff,g);

(%o26) DONE

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(%i27) a : ([1,2] ot [3,4,5]) ot ([6,5,4,3] ot [-1,2,-3,4,-5]);
(%o27) [[[[-18, 36, -54, 72, -90], [-15, 30, -45, 60, -75], [-12, 24, -36, 48, -60], [-9, 18, -27, 36, -45]], [[-24, 48, -72, 96, -120], [-20, 40, -60, 80, -100], [-16, 32, -48, 64, -80], [-12, 24, -36, 48, -60]], [[-30, 60, -90, 120, -150], [-25, 50, -75, 100, -125], [-20, 40, -60, 80, -100], [-15, 30, -45, 60, -75]], [[-36, 72, -108, 144, -180], [-30, 60, -90, 120, -150], [-24, 48, -72, 96, -120], [-18, 36, -54, 72, -90]], [[-48, 96, -144, 192, -240], [-40, 80, -120, 160, -200], [-32, 64, -96, 128, -160], [-24, 48, -72, 96, -120]], [[-60, 120, -180, 240, -300], [-50, 100, -150, 200, -250], [-40, 80, -120, 160, -200], [-30, 60, -90, 120, -150]]]]
(%i28) tensortype(a);
(%o28) [2,3,4,5]
(%i29) b : [x,y];
(%o29) [x, y]
(%i30) c : a ot b;

(%o30) [[[[[-18 x, -18 y], [36 x, 36 y], [-54 x, -54 y], [72 x, 72 y], [-90 x, -90 y]], [[-15 x, -15 y], [30 x, 30 y], [-45 x, -45 y], [60 x, 60 y], [-75 x, -75 y]], [[-12 x, -12 y], [24 x, 24 y], [-36 x, -36 y], [48 x, 48 y], [-60 x, -60 y]], [[-9 x, -9 y], [18 x, 18 y], [-27 x, -27 y], [36 x, 36 y], [-45 x, -45 y]], [[-24 x, -24 y], [48 x, 48 y], [-72 x, -72 y], [96 x, 96 y], [-120 x, -120 y]], [[-20 x, -20 y], [40 x, 40 y], [-60 x, -60 y], [80 x, 80 y], [-100 x, -100 y]], [[-16 x, -16 y], [32 x, 32 y], [-48 x, -48 y], [64 x, 64 y], [-80 x, -80 y]], [[-12 x, -12 y], [24 x, 24 y], [-36 x, -36 y], [48 x, 48 y], [-60 x, -60 y]], [[-30 x, -30 y], [60 x, 60 y], [-90 x, -90 y], [120 x, 120 y], [-150 x, -150 y]], [[-25 x, -25 y], [50 x, 50 y], [-75 x, -75 y], [100 x, 100 y], [-125 x, -125 y]], [[-20 x, -20 y], [40 x, 40 y], [-60 x, -60 y], [80 x, 80 y], [-100 x, -100 y]], [[-15 x, -15 y], [30 x, 30 y], [-45 x, -45 y], [60 x, 60 y], [-75 x, -75 y]], [[-36 x, -36 y], [72 x, 72 y], [-108 x, -108 y], [144 x, 144 y], [-180 x, -180 y]], [[-30 x, -30 y], [60 x, 60 y], [-90 x, -90 y], [120 x, 120 y], [-150 x, -150 y]], [[-24 x, -24 y], [48 x, 48 y], [-72 x, -72 y], [96 x, 96 y], [-120 x, -120 y]], [[-18 x, -18 y], [36 x, 36 y], [-54 x, -54 y], [72 x, 72 y], [-90 x, -90 y]], [[-48 x, -48 y], [96 x, 96 y], [-144 x, -144 y], [192 x, 192 y], [-240 x, -240 y]], [[-40 x, -40 y], [80 x, 80 y], [-120 x, -120 y], [160 x, 160 y], [-200 x, -200 y]], [[-32 x, -32 y], [64 x, 64 y], [-96 x, -96 y], [128 x, 128 y], [-160 x, -160 y]], [[-24 x, -24 y], [48 x, 48 y], [-72 x, -72 y], [96 x, 96 y], [-120 x, -120 y]], [[-60 x, -60 y], [120 x, 120 y], [-180 x, -180 y], [240 x, 240 y], [-300 x, -300 y]], [[-50 x, -50 y], [100 x, 100 y], [-150 x, -150 y], [200 x, 200 y], [-250 x, -250 y]], [[-40 x, -40 y], [80 x, 80 y], [-120 x, -120 y], [160 x, 160 y], [-200 x, -200 y]], [[-30 x, -30 y], [60 x, 60 y], [-90 x, -90 y], [120 x, 120 y], [-150 x, -150 y]]]]
(%i31) tensortype(c);
(%o31) [2,3,4,5,2]
(%i32) d : exchnext(c, 4)$
(%i33) tensortype(d);

(%o33) [2,3,4,2,5]
(%i34) f : exchnext(d, 3)$
(%i35) tensortype(f);

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(%o44) [[[[-18 x, -18 y], [-24 x, -24 y], [-30 x, -30 y]], [[-15 x, -15 y], [-20 x, -20 y], [-25 x, -25 y]], [[-12 x, -12 y], [-16 x, -16 y], [-20 x, -20 y]], [[-9 x, -9 y], [-12 x, -12 y], [-15 x, -15 y]]], [[[36 x, 36 y], [48 x, 48 y], [60 x, 60 y]], [[30 x, 30 y], [40 x, 40 y], [50 x, 50 y]], [[24 x, 24 y], [32 x, 32 y], [40 x, 40 y]], [[18 x, 18 y], [24 x, 24 y], [30 x, 30 y]]], [[[[-54 x, -54 y], [-72 x, -72 y], [-90 x, -90 y]], [[-45 x, -45 y], [-60 x, -60 y], [-75 x, -75 y]], [[-36 x, -36 y], [-48 x, -48 y], [-60 x, -60 y]], [[-27 x, -27 y], [-36 x, -36 y], [-45 x, -45 y]]], [[[72 x, 72 y], [96 x, 96 y], [120 x, 120 y]], [[60 x, 60 y], [80 x, 80 y], [100 x, 100 y]], [[48 x, 48 y], [64 x, 64 y], [80 x, 80 y]], [[36 x, 36 y], [48 x, 48 y], [60 x, 60 y]], [[[-90 x, -90 y], [-120 x, -120 y], [-150 x, -150 y]], [[-75 x, -75 y], [-100 x, -100 y], [-125 x, -125 y]], [[-60 x, -60 y], [-80 x, -80 y], [-100 x, -100 y]], [[-45 x, -45 y], [-60 x, -60 y], [-75 x, -75 y]]], [[[[-36 x, -36 y], [-48 x, -48 y], [-60 x, -60 y], [-30 x, -30 y], [-40 x, -40 y], [-50 x, -50 y]], [[-24 x, -24 y], [-32 x, -32 y], [-40 x, -40 y]], [[-18 x, -18 y], [-24 x, -24 y], [-30 x, -30 y]]], [[[72 x, 72 y], [96 x, 96 y], [120 x, 120 y]], [[60 x, 60 y], [80 x, 80 y], [100 x, 100 y]], [[48 x, 48 y], [64 x, 64 y], [80 x, 80 y]], [[36 x, 36 y], [48 x, 48 y], [60 x, 60 y]], [[[-108 x, -108 y], [-144 x, -144 y], [-180 x, -180 y]], [[-90 x, -90 y], [-120 x, -120 y], [-150 x, -150 y], [-72 x, -72 y], [-96 x, -96 y], [-120 x, -120 y], [[-54 x, -54 y], [-72 x, -72 y], [-90 x, -90 y]]], [[[144 x, 144 y], [192 x, 192 y], [240 x, 240 y]], [[120 x, 120 y], [160 x, 160 y], [200 x, 200 y]], [[96 x, 96 y], [128 x, 128 y], [160 x, 160 y]], [[72 x, 72 y], [96 x, 96 y], [120 x, 120 y]], [[-180 x, -180 y], [-240 x, -240 y], [-300 x, -300 y], [[-150 x, -150 y], [-200 x, -200 y], [-250 x, -250 y]], [[-120 x, -120 y], [-160 x, -160 y], [-200 x, -200 y]], [[-90 x, -90 y], [-120 x, -120 y], [-150 x, -150 y]]]]]
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(%i45) tensorType(%o44);
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(%o45) [2, 5, 4, 3, 2]
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```
(%i46) zerotensor([4,3,2]);
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(%o46) [[[0, 0], [0, 0], [0, 0]], [[0, 0], [0, 0], [0, 0]], [[0, 0], [0, 0], [0, 0]], [[0, 0], [0, 0], [0, 0]]]
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(%i47) kill(ALL);
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(%o0) DONE
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(%i1)
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